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MECHANICS.

229. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Find the position of the center of pressure of a semi-elliptical area completely immersed in water, the area being vertical, the bounding axis major being inclined to the horizon at an angle β , and having one extremity in the surface of the water.

Solution by the PROPOSER.

Let AB be the intersection of the vertical plane with the surface of the water; AC the major axis of the ellipse $=2a$; Q any point on the area of the semi-ellipse; $QD=h$, the perpendicular distance from Q to AB ; $\angle BAC=\beta$, $\angle CAQ=\theta$, $AQ=r$. Then

$$r = \frac{2ab^2 \cos \theta}{a^2 \sin^2 \theta + b^2 \cos^2 \theta}, \quad h = r \sin(\theta + \beta).$$

$$\bar{x} = \frac{\int_0^{\frac{1}{2}\pi} \int_0^r h r^2 \cos \theta \, d\theta \, dr}{\int_0^{\frac{1}{2}\pi} \int_0^r h r \, d\theta \, dr}, \quad \bar{y} = \frac{\int_0^{\frac{1}{2}\pi} \int_0^r h r^2 \sin \theta \, d\theta \, dr}{\int_0^{\frac{1}{2}\pi} \int_0^r h r \, d\theta \, dr}$$

$$\int_0^{\frac{1}{2}\pi} \int_0^r h r \, d\theta \, dr = \int_0^{\frac{1}{2}\pi} \int_0^r r^2 \sin(\theta + \beta) \, d\theta \, dr = \frac{1}{3} \int_0^{\frac{1}{2}\pi} r^3 \sin(\theta + \beta) \, d\theta = N.$$

$$\therefore N = \frac{8a^3 b^6}{3} \int_0^{\frac{1}{2}\pi} \frac{(\cos^3 \theta \sin \theta \cos \beta + \cos^4 \theta \sin \beta) \, d\theta}{(a^2 \sin^2 \theta + b^2 \cos^2 \theta)^3}$$

$$= \frac{1}{8} ab(4b \cos \beta + 3a \pi \sin \beta).$$

$$M = \int_0^{\frac{1}{2}\pi} \int_0^r h r^2 \cos \theta \, d\theta \, dr = \int_0^{\frac{1}{2}\pi} \int_0^r r^3 \cos \theta \sin(\theta + \beta) \, d\theta \, dr$$

$$= \frac{1}{4} \int_0^{\frac{1}{2}\pi} r^4 \cos \theta \sin(\theta + \beta) \, d\theta$$

$$= 4a^4 b^8 \int_0^{\frac{1}{2}\pi} \frac{(\cos^5 \theta \sin \theta \cos \beta + \cos^6 \theta \sin \beta) \, d\theta}{(a^2 \sin^2 \theta + b^2 \cos^2 \theta)^4}$$

$$= \frac{1}{24} a^2 b(16b \cos \beta + 15a \pi \sin \beta).$$

$$P = \int_0^{\frac{1}{2}\pi} \int_0^r h r^2 \sin \theta \, d\theta \, dr = \int_0^{\frac{1}{2}\pi} \int_0^r r^3 \sin \theta \sin(\theta + \beta) \, d\theta \, dr$$

$$\begin{aligned}
&= \frac{1}{4} \int_0^{\frac{1}{2}\pi} r^4 \sin \theta \sin(\theta + \beta) d\theta \\
&= 4a^4 b^3 \int_0^{\frac{1}{2}\pi} \frac{(\cos^4 \theta \sin^2 \theta \cos \beta + \cos^5 \theta \sin \theta \sin \beta) d\theta}{(a^2 \sin^2 \theta + b^2 \cos^2 \theta)^4}, \\
&= \frac{1}{2} ab^3 (16a \sin \beta - 3\pi b \cos \beta).
\end{aligned}$$

$$\bar{x} = M/N = \frac{1}{4} a \cdot \frac{16b \cos \beta + 15a \pi \sin \beta}{4b \cos \beta + 3a \pi \sin \beta}, \quad \bar{y} = P/N = \frac{1}{4} b \cdot \frac{16a \sin \beta + 3b \pi \cos \beta}{4b \cos \beta + 3a \pi \sin \beta}.$$

If $\beta=0$, $\bar{x}=a$, $\bar{y}=3\pi b/16$.

Also solved by S. Lefschetz.

230. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

A particle is projected from a distance $a=2r$ from the earth's center towards the earth with a velocity from infinity. If the earth were an airless homogeneous sphere, radius equal to the present mean radius and gravity as at present, with what velocity and in what time would it reach the center through an opening from surface to center?

Solution by the PROPOSER.

$$\int_{v_1}^v v dv = \int_x^a f dx, \text{ but } f = \frac{gr^2}{x^2} \text{ above the earth where } r \text{ is earth's radius.}$$

$$\therefore \int_{v_1}^v v dv = gr^2 = \int_x^a \frac{dx}{x^2}. \quad \therefore \frac{1}{2}(v^2 - v_1^2) = \frac{gr^2}{ax} (a-x).$$

$$\therefore v^2 = \frac{2gr^2}{ax} (a-x) + v_1^2 = \left(\frac{dx}{dt} \right)^2, \text{ where } v_1 \text{ is the initial velocity.}$$

$$\therefore t = \int_r^a \frac{1/\sqrt{x} dx}{\sqrt{[2agr^2 + x(av_1^2 - 2gr^2)]}}.$$

$$\text{If } v_1 = r\sqrt{\frac{2g}{a}}, \quad t = \frac{2(a^{\frac{3}{2}} - r^{\frac{3}{2}})}{3r_1\sqrt{(2g)}}, \quad v = \sqrt{(2gr)} \text{ at surface.}$$

$$\begin{aligned}
&\text{If } v_1 < r\sqrt{\frac{2g}{a}}, \quad t = 2gr^2 \sqrt{\left(\frac{a}{2gr^2 - av_1^2} \right)^3} \left(\cos^{-1} \sqrt{\frac{2gr^2 - av_1^2}{2agr}} \right. \\
&\quad \left. - \cos^{-1} \sqrt{\frac{2gr^2 - av_1^2}{2gr^2}} + \frac{1/\sqrt{(2gr^2 - av_1^2)} (2agr + av_1^2 - 2gr^2)}{2agr} \right)
\end{aligned}$$